

Problem 42: Geometry

(✓✓)

In the triangle ABC , angle $BAC = \alpha$ and angle $CBA = 2\alpha$, where 2α is acute, and $BC = x$. Show that $AB = (3 - 4\sin^2 \alpha)x$.

The point D is the midpoint of AB and the point E is the foot of the perpendicular from C to AB . Find an expression for DE in terms of x .

The point F lies on the perpendicular bisector of AB and is a distance x from C . The points F and B lie on the same side of the line through A and C . Show that the line FC trisects the angle ACB .

2015 Paper II

Comments

I thought it would be good to include at least one plane geometry question in this collection, so here it is. As always with geometry, the first thing to do is draw a BIG diagram. You will probably need to have quite a few tries at it.

Then you have the usual tools at your disposal: similar triangles, congruent triangles, angle-chasing, and — moving away from classical Euclidean geometry — Pythagoras, and sine and cosine rules. I leave it to you to decide what will be useful here.

